

A note of clarification: BICEP2 and Planck are not in tension

Benjamin Audren,¹ Daniel G. Figueroa,² and Thomas Tram¹

¹*Institut de Théorie des Phénomènes Physiques, École Polytechnique Fédérale de Lausanne, CH-1015, Lausanne, Switzerland*

²*Département de Physique Théorique and Center for Astroparticle Physics, Université de Genève, 24 quai Ernest Ansermet, CH1211 Genève 4, Switzerland*

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The apparent discrepancy between the value of the tensor-to-scalar ratio reported by the BICEP2 collaboration, $r = 0.20^{+0.07}_{-0.05}$ at 68% CL, and the Planck upper limit, $r < 0.11$ at 95% CL, has attracted a great deal of attention. In this short note, we show that this discrepancy is mainly due to an ‘apples to oranges’ comparison. The result reported by BICEP2 was measured at a pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$, assuming $n_t = 0$, whereas the Planck limit was provided at $k_* = 0.002 \text{ Mpc}^{-1}$, assuming the slow-roll consistency relation $n_t = -r/8$. One should obviously compare the BICEP2 and Planck results under the same circumstances. By imposing $n_t = 0$, the Planck constraint at $k_* = 0.05 \text{ Mpc}^{-1}$ becomes $r < 0.135$ at 95% CL, which can be compared directly with the BICEP2 result. Once a plausible dust contribution to the BICEP2 signal is taken into account (DDM2 model), r is reduced to $r = 0.16^{+0.06}_{-0.05}$ and the discrepancy becomes of order 1.3σ only.

THE PIVOT SCALE CONFUSION

A generic prediction of the inflationary paradigm is that the primordial scalar and tensor power spectra from inflation are nearly scale invariant. The deviation from scale invariance is then quantified by specifying a spectral index n and possibly a running α at a pivot scale k_* . The primordial power spectra are then given by

$$\mathcal{P}_s(k) = A_s \left(\frac{k}{k_*} \right)^{(n_s-1) + \frac{1}{2}\alpha_s \log\left(\frac{k}{k_*}\right)} \quad (1)$$

$$\mathcal{P}_t(k) = A_t \left(\frac{k}{k_*} \right)^{n_t + \frac{1}{2}\alpha_t \log\left(\frac{k}{k_*}\right)} \quad (2)$$

where $\alpha_s \equiv dn_s/d\log k$ and $\alpha_t \equiv dn_t/d\log k$. The primordial tensor amplitude A_t is, by convention, always substituted for the ratio

$$r \equiv \frac{\mathcal{P}_t(k_r)}{\mathcal{P}_s(k_r)}, \quad (3)$$

evaluated at a given scale k_r , which is often, but not always, taken to be k_* . The analysis of the temperature anisotropies by the Planck collaboration [1] was done at a pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$, but their reported constraint $r < 0.11$ at 95% Confidence Level (CL) was given at $k_r = 0.002 \text{ Mpc}^{-1}$. The BICEP2 collaboration [2], on the other hand, reported the amplitude $r = 0.20^{+0.07}_{-0.05}$ at 68% CL, evaluated at the Planck pivot scale¹ $k_r = k_* = 0.05 \text{ Mpc}^{-1}$. To avoid any confusion, it is then convenient to denote the tensor-to-scalar ratios evaluated at $k_r = 0.05 \text{ Mpc}^{-1}$ and $k_r = 0.002 \text{ Mpc}^{-1}$ by $r_{0.05}$ and $r_{0.002}$ respectively. That BICEP2 indeed has

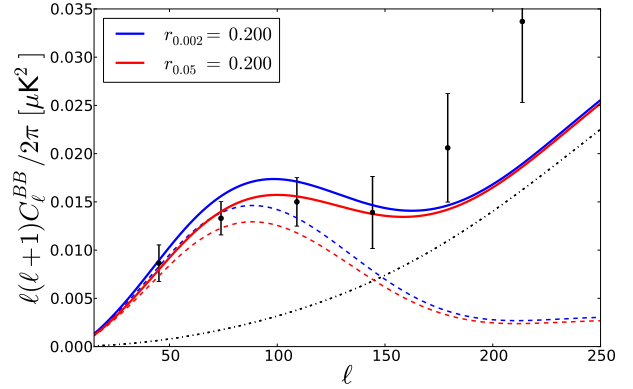


FIG. 1: B-mode angular power spectra for $r_{0.05} = 0.2$ (red curves) and $r_{0.002} = 0.2$ (blue curves), both for $n_t = \alpha_t = \alpha_s = 0$. The data points and error bars are from the table available at bicepkeck.org, and we have assumed Planck best-fit values for other cosmological parameters, in particular $A_s = 2.215 \times 10^{-9}$ and $n_s = 0.9619$. The dashed lines correspond to the B-modes from inflationary tensors, the dotted-dashed line show the contribution from the lensing of the (predominantly scalar) E-modes, and the continuous lines are the sum of the two.

$r_{0.05} = 0.2$ as best fit becomes evident in Fig. 1, where the low- ℓ B-mode angular power spectrum are plotted for $r_{0.05} = 0.2$ (red lines) and $r_{0.002} = 0.2$ (blue lines), considering for both cases $n_t = 0$, and assuming $\alpha_s = \alpha_t = 0$. The $r_{0.002} = 0.2$ curve is clearly not a good fit to the data points given by BICEP2.

THE LIKELIHOOD CONFUSION

The confusion related to the scale k_r is enhanced by the following circumstances: *i)* the BICEP2 collaboration used a different likelihood in their own analyses than

¹ Note that this is not explicitly stated in [2], at least not in the current arXiv version at the time of writing this note, but it has been confirmed to us by the BICEP2 collaboration.

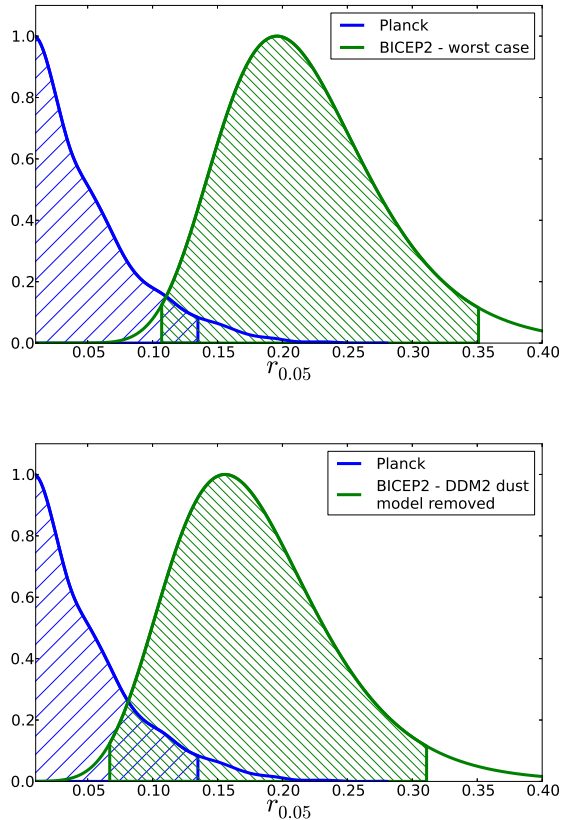


FIG. 2: Posteriors for $r_{0.05}$ from both Planck (blue) and BICEP2 (green). For BICEP2, in the top panel, we plot the tabulated likelihood available from bicepkeck.org, obtained assuming Planck best-fit values for the remaining cosmological parameters; in the bottom panel, we shift the same distribution by -0.04 , to account approximatively for dust removal assuming the plausible DDM2 model discussed by the collaboration. For Planck, we show the full posterior distribution obtained with a Markov Chain Monte Carlo sampling of the 6 Standard Model parameters and $r_{0.05}$, as well as the 14 nuisance parameters of Planck, assuming $n_t = \alpha_t = \alpha_s = 0$. The vertical lines encapsulate the 2σ allowed regions for each distribution.

the publicly released Python likelihood code, and *ii*) the best-fit of the public code is actually $r_{0.002} = 0.2$, and *not* $r_{0.05} = 0.2$ as found in the BICEP2 analysis. This difference in best-fit is due to two separate facts: *a*) different methods are being used for computing the likelihood, the public one using the Hamimeche & Lewis code, whereas the private one uses the formula introduced in [3], paragraph 9.3.1, and *b*) the public code uses information from all nine bandpower bins, whereas the internal one makes use of only the five first ones.

It should be noted that the difference between the best-fit values of the two likelihoods is well below 1σ . So this is not alarming in any way, but it leads nonetheless to an overestimation of the tension with Planck when using

the public code.

In conclusion, the only data product matching exactly the BICEP2 internal analysis is the tabulated likelihood, obtained for a fixed cosmology with different values of $r_{0.05}$, represented in green in the top panel of Fig. 2. This corresponds to the advertised value of $r_{0.05} = 0.20^{+0.07}_{-0.05}$. Reference [2] also discusses several dust models, retaining DDM2 (Data Driven Model 2) as the most plausible one. After removing the DDM2 contamination, the BICEP2 collaboration obtains $r_{0.05} = 0.16^{+0.06}_{-0.05}$ (68% CL). In the lower panel of Fig. 2 we present an approximative r posterior after dust removal.

COMPARISON WITH PLANCK

Using Eqs. (1) and (2), one can convert $r_{0.05}$ from BICEP2 to $r_{0.002}$ for comparison with Planck, or vice versa. For instance, the $r_{0.05} = 0.2$ curve in Fig. 1, which is the best fit to the data (for $n_t = \alpha_t = \alpha_s = 0$), is equivalent to $r_{0.002} \simeq 0.177$. However, this would still be an ‘apples to oranges’ comparison, since the Planck analysis used a tensor spectral index inferred from the single-field slow-roll consistency condition $n_t = -r/8$, while BICEP2 used $n_t = 0$. This means that the underlying tensor primordial spectra was not of the same form, so it is in principle meaningless to compare the two parameters: If one experiment fits $y = a_0 + a_1x$ to the data while the other fits $y = b_0$, we certainly should not compare a_0 and b_0 .

We derived the posterior probability for $r_{0.05}$ assuming a flat Λ CDM+ r model and the Planck+WP dataset. In any Bayesian parameter extraction, the posterior depends on the choice of prior. Here, we choose to restrict ourselves to physical models by imposing a prior $r_{0.05} \geq 0$ (a different choice is advocated in the recent analysis of [4]). After running the CLASS and MONTE PYTHON codes, we obtained $r_{0.05} < 0.135$ at 95% CL, which is not in significant tension with the BICEP2 result, as shown in Fig. 2. Even before subtracting the dust model, the two posteriors overlap at the 9% CL (corresponding to 1.7σ). After removing dust contamination (under the DDM2 assumption), the compatibility increases² to the level of 17%, corresponding to a 1.3σ overlap.

With such an overlap between the two likelihoods, we can conclude (even without calculating Bayesian evidence ratios) that there is no compelling reason at the moment to invoke extra ingredients in the cosmological model, in order to alleviate a would-be tension between

² Here the compatibility is quantified by searching for the confidence level of each likelihood above which there is an overlap. Another statistical test of the compatibility between two such likelihoods is presented in [4].

the Planck 2013 and BICEP2 measurements. In particular, there is no convincing case for introducing a non-zero scalar running α_s of the order of -0.02 , which would be incompatible with the simplest and most elegant slow-roll inflationary paradigm.

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- [1] P. Ade et al. (Planck Collaboration) (2013), 1303.5076.
 - [2] P. Ade et al. (BICEP2 Collaboration) (2014), 1403.3985.
 - [3] D. Barkats et al. (BICEP1 Collaboration) (2013), 1310.1422.
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